THE KONTSEVICH GRAPH COMPLEX, OR HOW YOU'LL SUCCEED ON DEFORMING MATH WOR(L)DS

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On $\mathbb{R}^{\geqslant 2}$ consider the commutative associative multiplication $\times$ in the unital algebra of smooth functions. How can the operation $\times$ be deformed to $\star = \times + \cdots$ in such a way that the $\star$-product remains associative but not necessarily commutative?

On $\mathbb{R}^{\geqslant 3}$ with Cartesian coordinates x^1, ..., x^n let there be a Poisson structure; for instance, take any $(n-2)$-tuple of smooth functions h_3, ..., h_n and for any smooth functions f and g consider the Jacobian determinant

$$\{f, g\} := \det\Bigl(\operatorname{Jac}\bigl(\partial(f, g, h_3, \ldots, h_n)/\partial(x^1, \ldots, x^n)\bigr)\Bigr).$$

How can the Poisson bracket at hand be deformed to $\{\cdot,\cdot\} + \cdots$ in such a way that it stays Poisson at least infinitesimally? (That is, the Jacobi identity remains valid up to higher powers of the deformation parameter.)

How are the above questions interrelated and indeed, why is there a relation between them?

The Kontsevich complex of oriented graphs is a mathematical language by using which all these questions are successfully answered. The talk will be an introduction to this classical domain where elementary formulations of hard open problems are still abundant.